Kittel

4.4. with $C_p = A \frac{\sin(pk_0 a)}{pa}$, $\omega^2 = \frac{2}{M} \sum_{p>0} C_p(1-\cos(pka))$,

we compute ω^2 explicitly:

$$\omega^2 = \frac{2A}{M} \sum_{p>0} \frac{\sin(pk_0 a)}{pa} \left[1 - \cos(pka)\right].$$

$$\boxed{\omega^2 = \frac{2A}{M} \sum_{p>0} \left\{ \frac{\sin pk_0 a}{pa} - \frac{1}{2}\sin[pa(k_0+k)] - \frac{1}{2}\sin[pa(k_0-k)] \right\}}$$

$$\frac{d\omega^2}{dk} = \frac{2A}{M} \sum_{p>0} \left\{ -\frac{1}{2}\cos[pa(k_0+k)](pa) - \frac{1}{2}\cos[pa(k_0-k)](-pa) \right\}$$

$$= \boxed{\frac{2A}{M} \sum_{p>0} \frac{pa}{2} \left\{ \cos[pa(k_0-k)] - \cos[pa(k_0+k)] \right\}}.$$

when $k = k_0$, the above expression becomes

$$\frac{d\omega^2}{dk} = \frac{A}{M} \sum_{p>0} pa \left\{ \cos[0] - \cos[2pak_0] \right\}$$

$$= \frac{A}{M} \sum_{p>0} pa \left\{ 1 - \cos[2pak_0] \right\}.$$

For this expression to converge, $\{1 - \cos[2pak_0]\}$ must at least go like p^{-2}, yet this is clearly false.